Shift-Variant Deblurring for Rotationally Symmetric Systems

Amit Kohli*, Anastasios Angelopoulos*, Sixian You, Laura Waller
Department of Electrical Engineering and Computer Sciences, University of California, Berkeley
*apkohli@berkeley.edu, * equal contribution*

Abstract: We present a fast image deblurring method for rotationally symmetric systems with spatially-varying aberrations. We calibrate our method with point spread function measurements along a line. Our method outperforms standard deconvolution on the UCLA Miniscope.

1. Introduction

The ubiquitous assumption that an optical system is *linear shift-invariant* (LSI) yields fast and simple deconvolution for image deblurring. However, for imaging systems that aspire to capture wide fields-of-view (FoVs) with simple and/or inexpensive optics, the LSI assumption often breaks down—that is, the system's *point spread function* (PSF) changes shape across the FoV. Thus, to describe this spatial variance, we must know the PSF at each point in the FoV. A naïve strategy for shift-variant deblurring involves collecting all of the PSFs and then inverting a large least squares problem. However, this has intractable calibration, memory, and compute requirements.

Our paper introduces a technique that achieves accurate image recovery for systems which are symmetric about the optical axis, while requiring dramatically fewer resources. Our method, called *linear revolution-invariant* (LRI) deblurring, exploits the fact that the PSF only changes shape when a point source moves radially in a rotationally symmetric system; hence, a single ray of calibration measurements fully characterizes the transfer function. Even for systems that are not exactly rotationally symmetric, this method may provide a good trade-off between image quality improvement and calibration/compute resources. We introduce a mathematical extension of the convolution theorem for LRI systems leading to computationally-efficient forward and inverse methods. We demonstrate that our method outperforms traditional deconvolution (assuming LSI) for image deblurring with realistic PSF data from a simulated miniature microscope system [1].

2. Theory

Computing the forward model (the final intensity image $I(x,y)$) of a linear optical system corresponds to a weighted sum of PSFs over all points (u,v), which concretely is $I(x,y) = \int\int f(u,v)h(x,y;u,v)dudv$, where $f(u,v)$ describes the object intensity at (u,v) and $h(x,y;u,v)$ describes the PSF generated by a point source at (u,v) [2]. In the LSI model, the shape of a PSF no longer varies with (u,v), so we can write $h(x,y;u,v) = h(x-u,y-v;0,0)$, making I a convolution that only depends on one PSF.

Instead, we work under the assumption that the system is rotationally symmetric; if the object rotates with respect to the optical axis, its image also rotates by the same amount (note how the PSFs rotate as the point source location is rotated in Fig. 1(b)). Specifically, define $\tilde{h}(\rho,\phi;r,\theta) = h(\rho\cos\phi,\rho\sin\phi;r\cos\theta,r\sin\theta)$ to be the expression of h in polar coordinates, and $\tilde{I}$, $\tilde{f}$ similarly. The system is *LRI* if $\tilde{h}(\rho,\phi;r,\theta) = \tilde{h}(\rho,\phi-\theta;r,0)$ which means the PSF shape only depends on the distance r of the point source from the optical axis. We drop the last argument of $\tilde{h}$ for convenience, writing $\tilde{h}(\rho,\phi-\theta;r) := \tilde{h}(\rho,\phi-\theta;r,0)$. Plugging the LRI assumption into the forward model yields the LRI Convolution theorem.

Theorem 1 (LRI Convolution theorem)**.** *Under the LRI assumption, where $\mathscr{F}_\theta$ is a Fourier transform over θ,*

$$\tilde{I}(\rho,\phi) = \int\int r\tilde{f}(r,\theta)\tilde{h}(\rho,\phi;r,\theta)d\theta dr = \int r\mathscr{F}_\theta^{-1}\left\{\mathscr{F}_\theta\{\mathbb{1}_{\{\theta\in(-\pi,\pi)\}}\tilde{f}(r,\theta)\}\mathscr{F}_\theta\{\tilde{h}(\rho,\theta;r)\}\right\}(\phi)dr. \quad (1)$$

Proof. Applying the LRI assumption to the left-hand side of (1) and replacing the inner integral with an indicator function yields a convolution over θ to which we apply the standard convolution theorem,

$$\tilde{I}(\rho,\phi) = \int_0^\infty r\int_{-\pi}^{\pi}\tilde{f}(r,\theta)\tilde{h}(\rho,\phi-\theta;r)d\theta dr = \int_0^\infty r\int_{-\infty}^{\infty}\tilde{f}(r,\theta)\mathbb{1}_{\{\theta\in(-\pi,\pi)\}}\tilde{h}(\rho,\phi-\theta;r)d\theta dr. \quad \blacksquare$$

3. Results

We operationalize the theorem above into LRI forward and inverse algorithms carried out in Python using PSF data obtained from a Zemax OpticsStudio simulation of Miniscope v3 [1], which will have shift-variant aberrations due to the miniature gradient-index (GRIN) objective, yet is a rotationally symmetric system by design (Fig. 1(a)). Figure 1(b) shows a set of PSFs from point sources in a radial grid; notice that they change identically for any angle as we move out radially within the FoV; hence, the PSF is LRI but not LSI.

We start with the forward models. The true measured intensity is calculated as a weighted sum of the PSFs at every point source location (u,v), where the weight is $f(u,v)$. As a standard baseline, we calculate the measurement predicted by the LSI forward model (by convolving the test image with the center PSF) in addition to our LRI forward model (by using a single radial line of PSFs and Eq. 1). Since the forward model is a key piece of the inverse solver, it is important for it to be computationally efficient. The LRI forward model requires both fewer PSFs than the true measurement, and is several orders-of-magnitude faster, even as the number of pixels grows (Fig. 1(c)). Of course, the LSI method is faster than our method and only uses one calibration PSF, but its error map shows significant departure from the true image, particularly near the edges of the FoV, Fig. 1(d).

Finally, we perform deblurring with each method by optimizing the input to the forward model that minimizes the L2 distance to the true measurement plus a small amount of anisotropic TV regularization for denoising. Figure 1(e) demonstrates that our LRI method provides sharper image reconstruction than LSI, especially at the edges of the FoV, which is consistent with the forward model error maps. Furthermore, LRI achieves a lower MSE and higher SSIM than LSI (0.003 vs. 0.016 and 0.88 vs. 0.56, respectively). In summary, our approach provides significant improvements in image quality, with minimal calibration and computation expense, and should find use in rotationally symmetric shift-varying systems like the Miniscope3D [3].

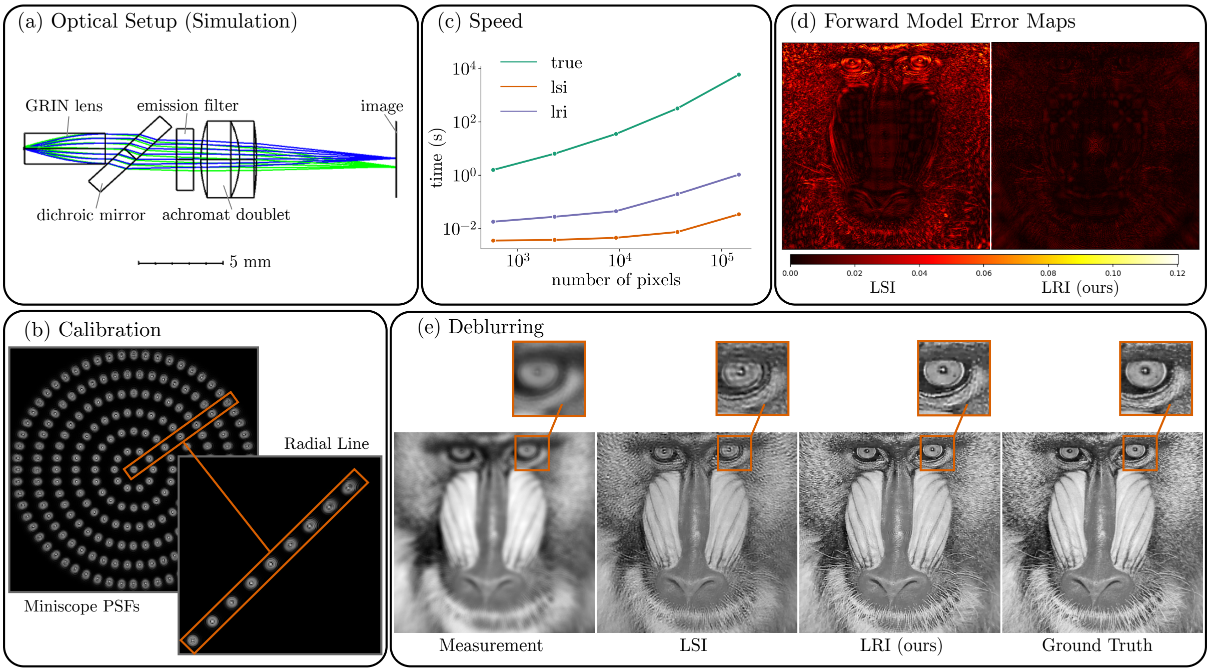

Fig. 1. (a) The Miniscope v3 in Zemax OpticsStudio. (b) PSFs from a polar grid of point sources across the FoV and (inset) PSFs from a single radial line of positions. (c) Speed of computation for each forward model as a function of the number of pixels in the test image. (d) Error maps showing the absolute difference from the true measurement for LSI and LRI forward models. (e) Deblurring results for each method, given the simulated input measurement with additive Gaussian noise ($\sigma = 0.001$) and compared to ground truth.

References

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